



# Discovering Universal Geometry in Embeddings with ICA

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**Purpose**

- Understanding How Embedding Geometry Encodes Meaning
- Exploring the Universality of Geometric-Meaning Relationships in Embeddings

**We use ICA**

**Independent Component Analysis (ICA)**  
Finds Statistically Independent Axes

**Results**

- Independent Axes in Embeddings are **"Spiky"** and **Interpretable**
- "Spiky"** and **Interpretable** Axes are Universal in Various Embeddings

**Raw (X)**

**PCA (XA)**

**ICA-transformed Embeddings (XAR)**

## Universal Geometry in Embeddings

**① Cross-Lingual**

English

Spanish

Chinese

**② Model**

Static

Contextualized

Vision

Language

**③ Modality**

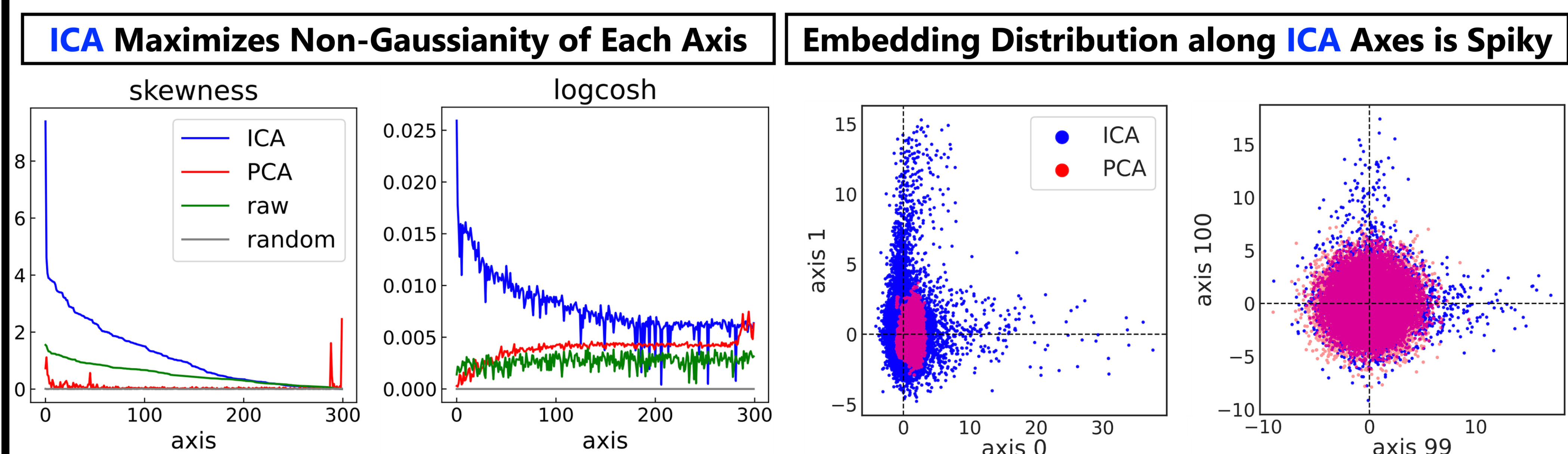
- Apply ICA to each embedding
- Permute axes by correlation for translation pairs

## ICA Discovers Spiky Axes

- Two steps for **ICA** to find independent axes
  - Whitening (PCA)**: Make each axis uncorrelated
  - Orthogonal Transformation**: Maximize the **non-Gaussianity** of each axis
- Embedding distribution along independent axes is **"spiky"**
  - PCA can't** find the "spiky" axes, providing **isotropic** embedding
  - ICA can** find the "spiky" axes by focusing on **anisotropic** information

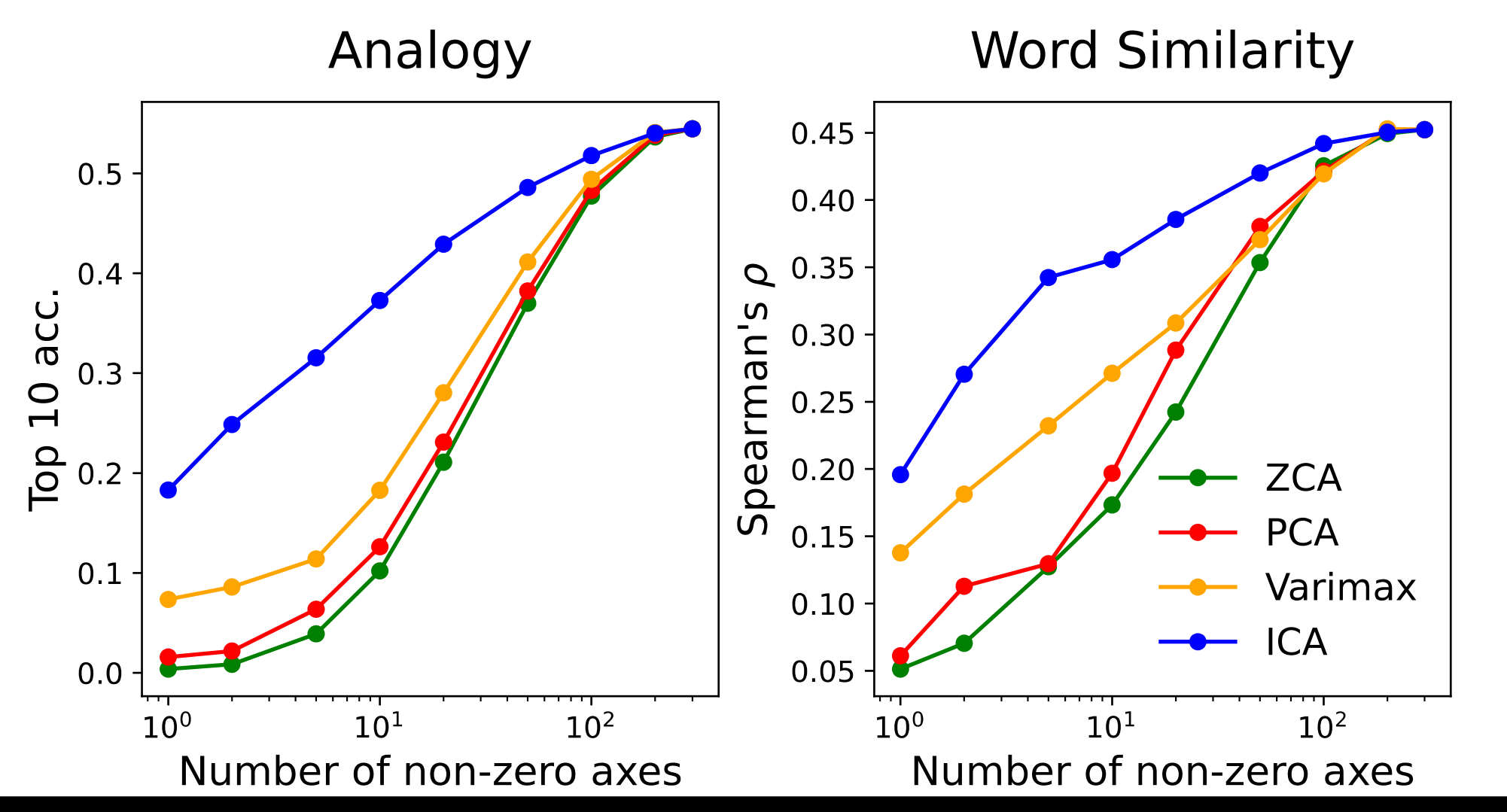
ICA-transformed

$$S = XAR$$



## Low-Dimensionality

- ICA-transformed embedding** represents meaning with **few axes**
- Evaluation Tasks
  - Analogy
  - Word Similarity
- Evaluate performance by **reducing the number of non-zero axes**



**Ref.** [1] Aapo Hyvärinen and Erkki Oja. Independent Component Analysis: Algorithms and Applications. (Neural Networks 2000) [2] Tomáš Musil and David Mareček. Independent Components of Word Embeddings Represent Semantic Features (arXiv 2022)